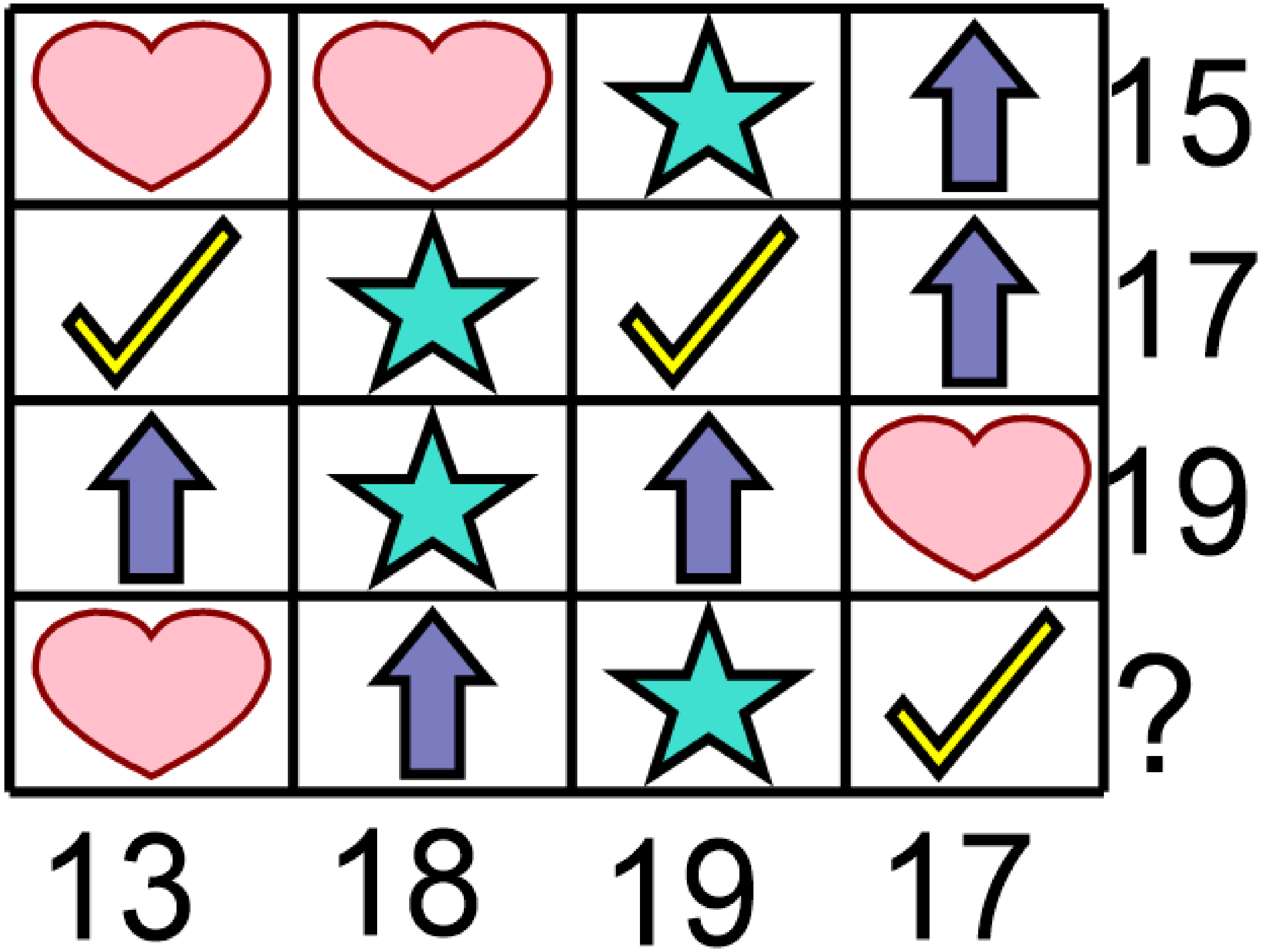
Further Maths

Induction Day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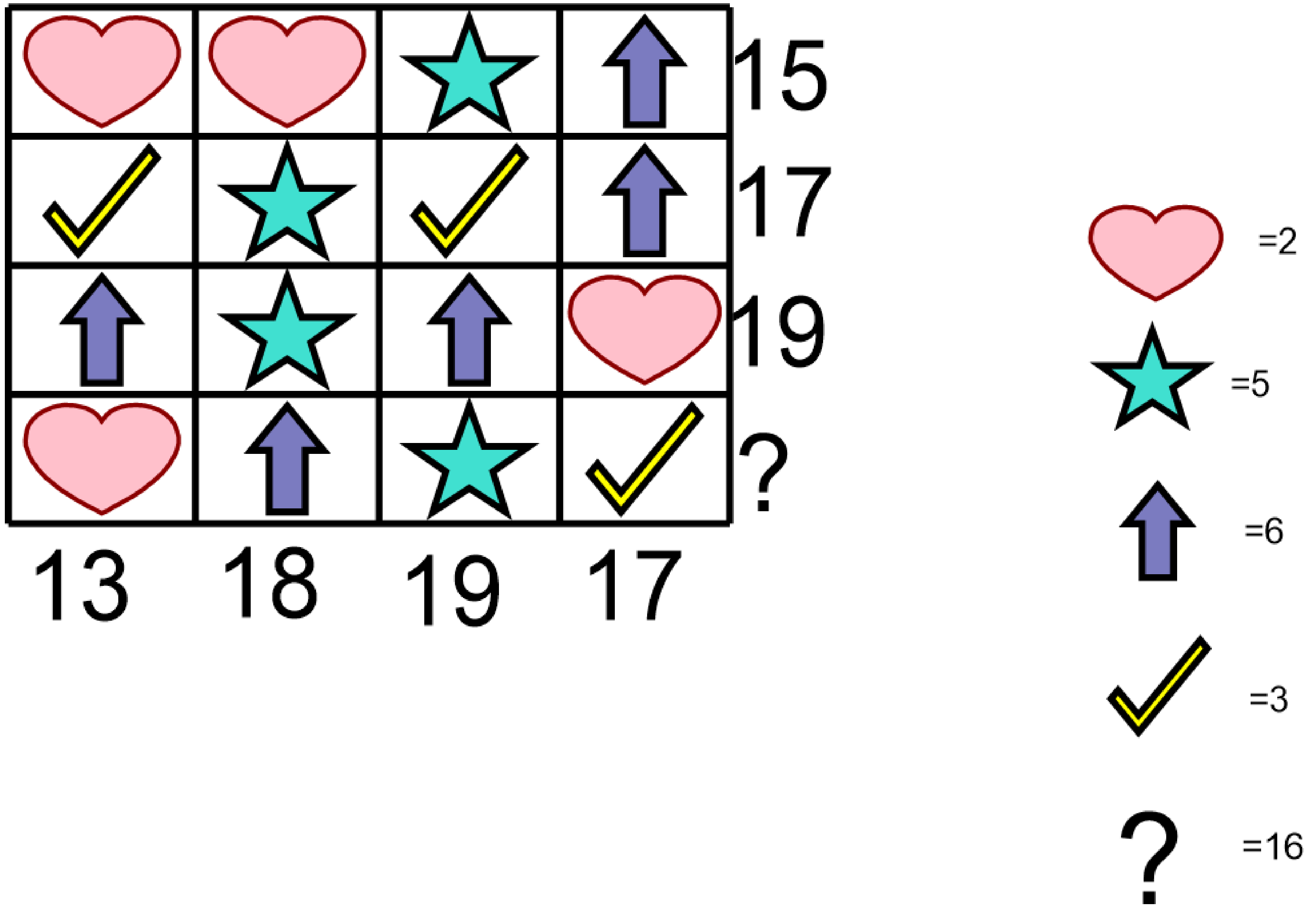
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=2
=5
=6
=3
=16

Can you spot the mistake in the following "proof" that 2+2=5?

$$-20=-20$$

$$16-36=25-45$$

$$(2+2)^2-(2+2)\times 9=5^2-5\times 9$$

$$(2+2)^2-2\times(2+2)\times\frac{9}{2}=5^2-2\times 5\times\frac{9}{2}$$

$$(2+2)^2-2\times(2+2)\times\frac{9}{2}+\left(\frac{9}{2}\right)^2=5^2-2\times 5\times\frac{9}{2}+\left(\frac{9}{2}\right)^2$$

$$\left(2+2-\frac{9}{2}\right)^2=\left(5-\frac{9}{2}\right)^2$$

$$2+2-\frac{9}{2}=5-\frac{9}{2}$$

$$2+2=5$$

The mistake lies in the penultimate step:

$$\left(2+2-\frac{9}{2}\right)^2=\left(5-\frac{9}{2}\right)^2$$
$$2+2-\frac{9}{2}=5-\frac{9}{2}$$

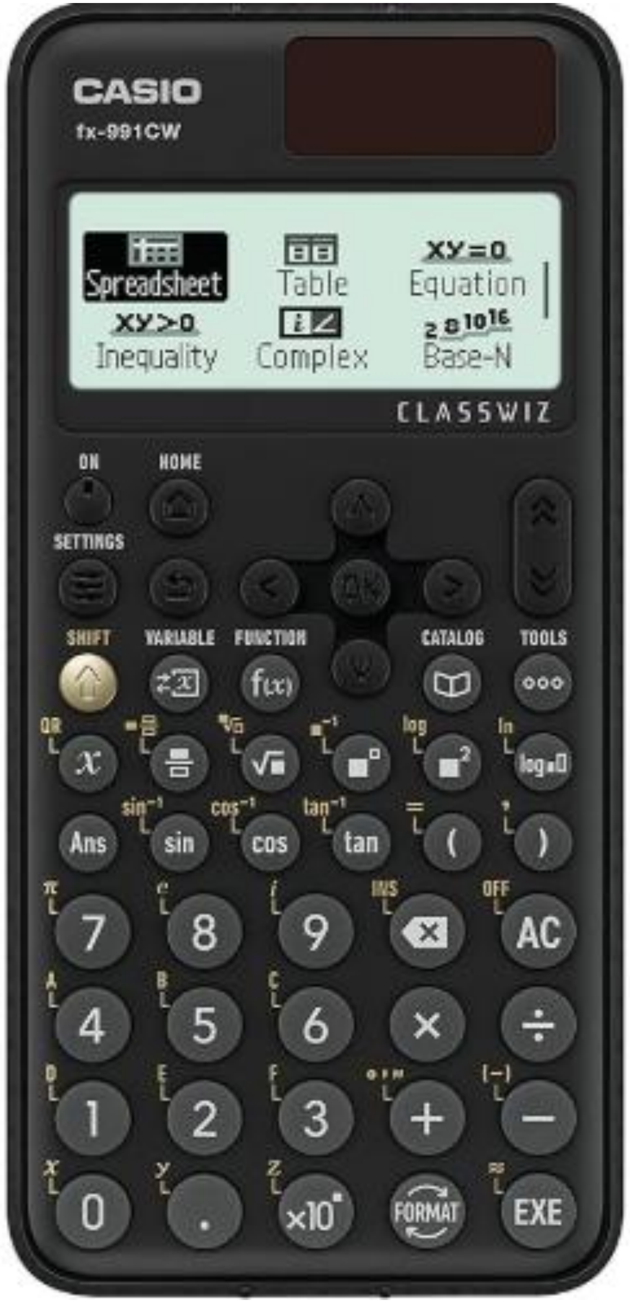
If we compare this to its simplified form:

In the 'proof':	Simplified:
$\left(2+2-\frac{9}{2}\right)^2=\left(5-\frac{9}{2}\right)^2$	$\left(-\frac{1}{2}\right)^2=\left(\frac{1}{2}\right)^2$
$2+2-\frac{9}{2}=5-\frac{9}{2}$	$-\frac{1}{2}=\frac{1}{2}$

So the mistake lies in the assumption that $x^2=y^2 \Rightarrow x=y$.

A few things before we start…

- How does the course work?
- Who will be teaching me?
- What resources do I need?
- What is the difference between Maths and Further Maths?
- What should I be doing over the summer?



New Calculator

Casio fx-991CW Advanced Scientific Calculator

These can be bought online or in TG Jones etc.

Amazon link:

[Casio fx-991CW Advanced Scientific Calculator (UK Version) : Amazon.co.uk: Stationery & Office Supplies]

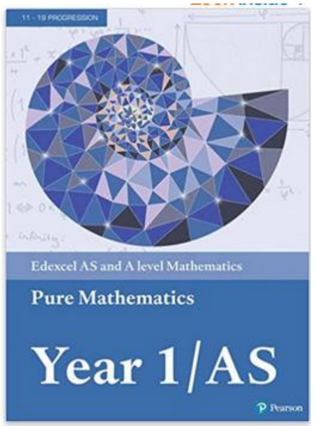
Watch out for fakes or very expensive versions!!!

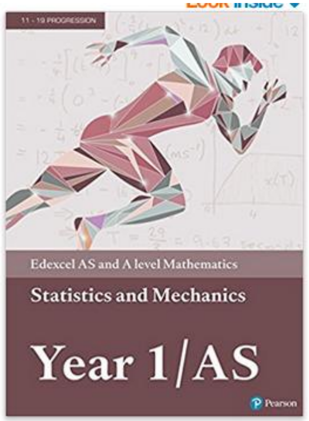
It should be around £25.

Textbooks

These are on Amazon but will sell out quickly in September.
If you are ordering gradually, order the Pure ones first.

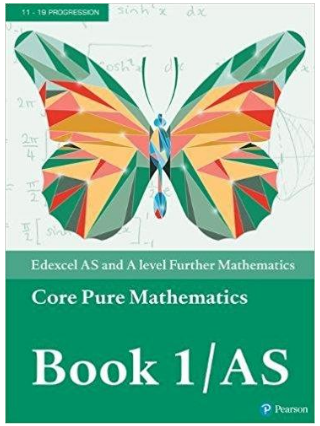
ISBNs as follows (just type these numbers into search on Amazon, other providers are available, and they should pop up):

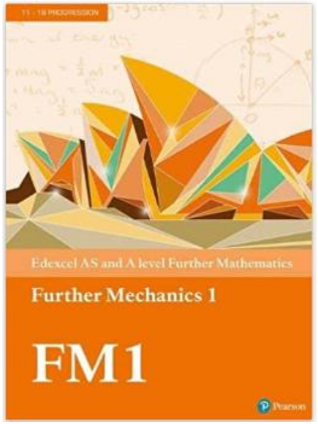


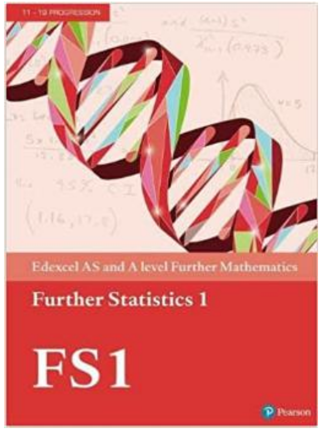


Regular Maths:

1292232536 – Statistics and Mechanics
129218339X – Pure



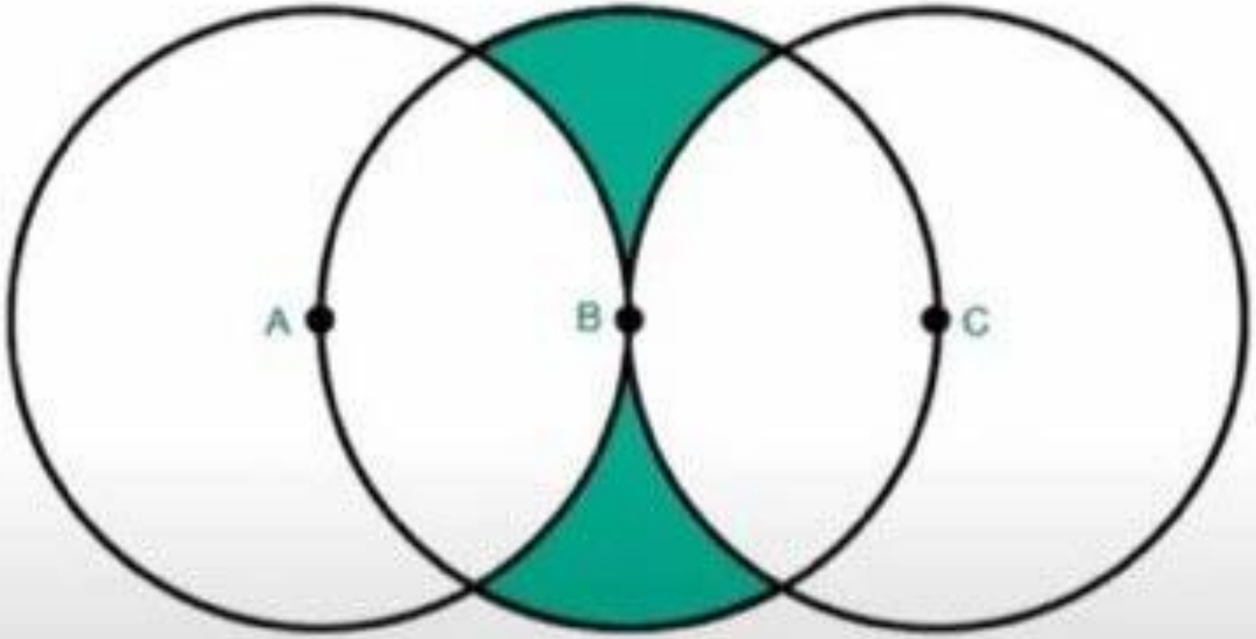




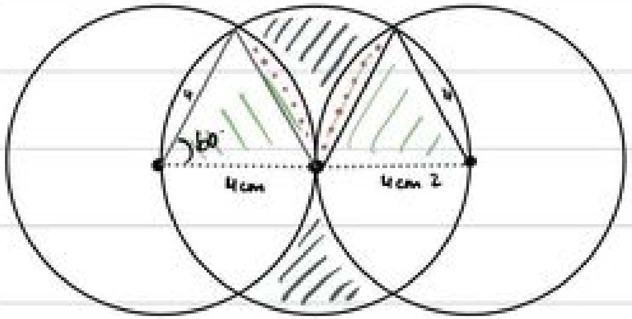
Further Maths:

1292183373 – Further Stats
1292183314 – Further Mechanics
1292183330 – Core Pure

The diagram is made of three circles, each of radius 4cm.
The centre of the circles are A, B and C, such that ABC is a straight line.



Work out the total area of the two shaded regions.
Give your answer in terms of π



NOTE: All three triangles have the same area, since they all have radius 4.

① Area of one circle $= \pi \times (4)^2 = 16\pi$. So, semi-circle area is 8π.

② Area of sector $= \frac{60}{360} \times \pi \times (4)^2 = \frac{8}{3}\pi$

Then, equilateral triangle area $= \frac{1}{2}(4)(4)\sin 60$ (60° since triangle is equilateral)

$= 8 \times \frac{\sqrt{3}}{2} = 4\sqrt{3}$.

So, area of $= \frac{8}{3}\pi - 4\sqrt{3}$

③ Hence, top shaded area $= 8\pi - 2\left(\frac{8}{3}\pi + \frac{8}{3}\pi - 4\sqrt{3}\right)$

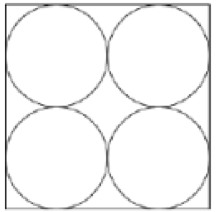
$= 8\pi - \frac{32}{3}\pi + 8\sqrt{3}$

$= \frac{24}{3}\pi - \frac{32}{3}\pi + 8\sqrt{3}$

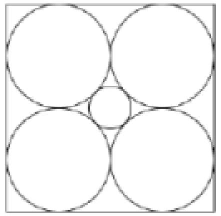
$= -\frac{8}{3}\pi + 8\sqrt{3}$

④ So, total shaded $= 2\left(-\frac{8}{3}\pi + 8\sqrt{3}\right) = -\frac{16}{3}\pi + 16\sqrt{3}$

The diagram shows four circles of radius ¼ in a unit square.



A circle can fit in the gap in the middle.



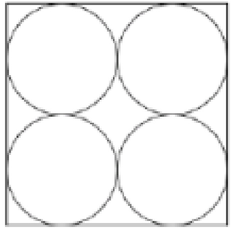
What is the radius of the circle in the middle? What proportion of the original square is covered by the circle in the middle?

If you placed eight spheres of radius ¼ in a unit cube a sphere can fit in the middle. What is the radius of the sphere in the middle? What proportion of the original cube is covered by the sphere in the middle?

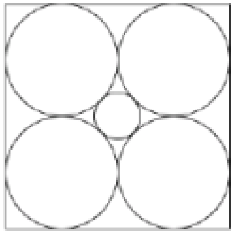
What happens in 4 dimensions?

And in higher dimensions how big does the diameter of the hypersphere in the gap get and what proportion of the hypercube is covered?

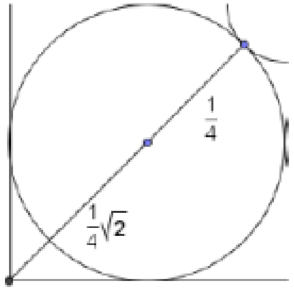
The diagram shows four circles of diameter ½ in a unit square.



A circle can fit in the gap in the middle.



This circle has diameter $\sqrt{2}-2\left(\frac{1}{4}\sqrt{2}+\frac{1}{4}\right)$



If you placed eight spheres of diameter ½ in a unit cube then you would find that a sphere of diameter $\sqrt{3}-2\left(\frac{1}{4}\sqrt{3}+\frac{1}{4}\right)$ would fit in the gap in the middle. This diameter is bigger than the diameter of the circle in the 2D version.

What happens in 4 dimensions?

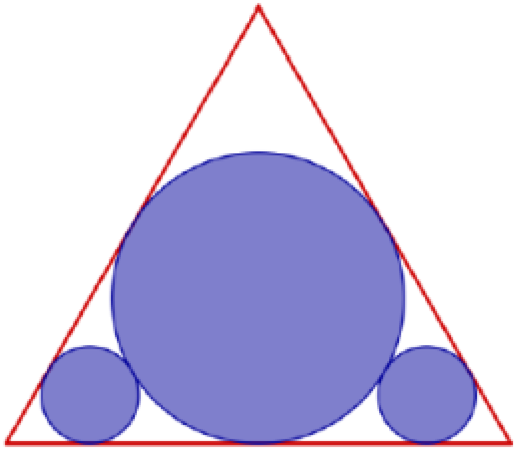
And in higher dimensions how big does the diameter of the hypersphere in the gap get?

In 9 dimensions the diameter of the hypersphere in the gap is $\sqrt{9}-2\left(\frac{1}{4}\sqrt{9}+\frac{1}{4}\right)=1$ and in 10 dimensions the diameter of the hypersphere in the gap is $\sqrt{10}-2\left(\frac{1}{4}\sqrt{10}+\frac{1}{4}\right)=\frac{\sqrt{10}-1}{2}>1$ – the hypersphere doesn’t fit inside the hypercube!

In an equilateral triangle side length 1 what is the largest area inside the triangle that can be covered by one circle?

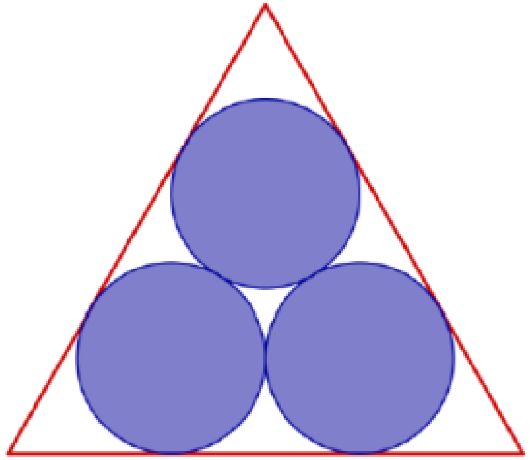
In an equilateral triangle side length 1 what is the largest area inside the triangle that can be covered by three, non-overlapping circles?

The maximum area can be achieved fitting the circle with largest area inside the triangle and then fitting two more in two of the corners:



This gives an area of $\frac{11\pi}{8}$ or 0.31997703 square units.

This is 1.4% larger than fitting three circles inside the triangle such that each circle is tangent to two sides of the triangle and the other two circles (these are known as Malfatti circles):



This has an area of $\frac{6-3\sqrt{3}}{8}\pi$ or 0.31567021 square units.

The fact that the Malfatti circles do not give the maximum area was only observed in 1930.

Over the summer

- Complete the circles sheet.
- Complete one (or both if you're bored/interested) of the courses below:

Egyptian Mathematics www.open.edu/openlearn/science-maths-technology/mathematics-statistics/egyptian-mathematics/?active-tab=description-tab

Babylonian Mathematics https://www.open.edu/openlearn/science-maths-technology/mathematics-statistics/babylonian-mathematics/?active-tab=description-tab